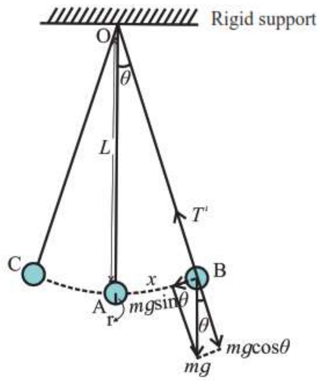
OSCILLATION PART 2

Simple Pendulum: An ideal simple pendulum is a heavy particle suspended by a massless, inextensible, flexible string from a rigid support.
Practically a small heavy dense sphere called bob, is suspended by a light and inextensible string from a rigid support



L: distance from the suspension and centre of gravity of the bob (length of pendulum)
m: mass of the bob
T' : Tension in the string at extreme position
θ: Small angle the pendulum is displaced by and then released
A: mean position

T' = mg cosθ

Restoring force F= –mgsinθ
Since θ is small, thus
sinθ≈θ≈x/L
Thus, F = -mgθ = - mgx/L
Thus Fα -x . **Hence the bob performs linear SHM for small amplitudes**

$$Time\ period\ T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{acceleration\ per\ unit\ displacement}} \quad ......(I)$$

$$Since, F = \frac{-mgx}{L}, \qquad Thus, ma = \frac{-mgx}{L}$$

$$Thus, \frac{a}{x} = \frac{g}{L}\ (in\ magnitude)$$

Substitute in (I) we get

$$T = \frac{2\pi}{\sqrt{g/L}} = 2\pi\sqrt{\frac{L}{g}} \quad and\ frequency\ n = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Laws of simple pendulum:

- The period is directly proportional to square root of its length
  $T\ \alpha\ \sqrt{L}$
- The period of a simple pendulum is inversely proportional to square root of gravity
  $T\ \alpha\ \frac{1}{\sqrt{g}}$
- The period does NOT depend on mass of the bob
- The period does NOT depend on the amplitude (if its small)

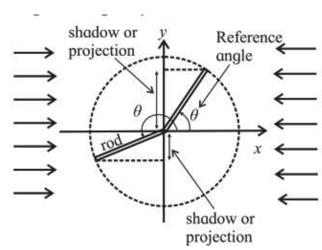
Second's Pendulum: A simple pendulum whose period is TWO seconds is called a second's pendulum

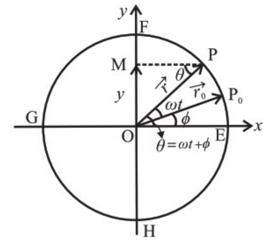
$$T = 2\pi\sqrt{\frac{L}{g}}, BUT\ T = 2s$$

$$2 = 2\pi\sqrt{\frac{L}{g}}$$

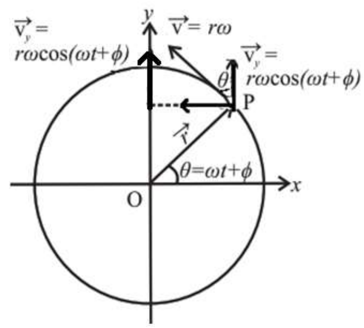
$$Thus, L = \frac{g}{\pi^2}$$

Reference Circle Method:





Consider a rod rotating in an anticlockwise direction and we are projecting the tip of the rod on to the y axis. This shadow or projection of the tip will be performing linear SHM along the y axis. Thus, we will show that the project of UCM is a SHM
Consider a point P performing UCM in anticlockwise direction about point O. Let Po be the initial point with ϕ as the initial phase (or epoch) and ωt being the angular displacement in time t. Thus, θ = ωt + ϕ
The projection of P on the y-axis is at M
Thus, y = OP sinθ = r sinθ = r sin(ωt + ϕ)
For velocity,

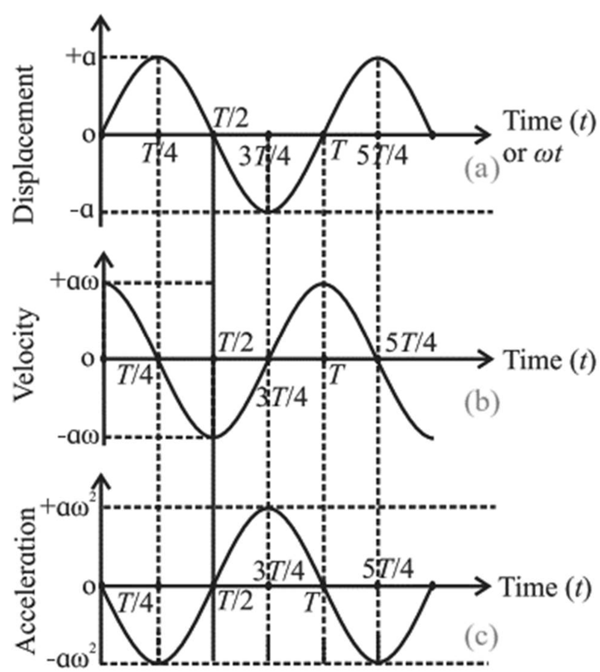


The velocity projection on y axis will be
v_y = v cosθ = rω cos θ
 = rω cos(ωt + ϕ)

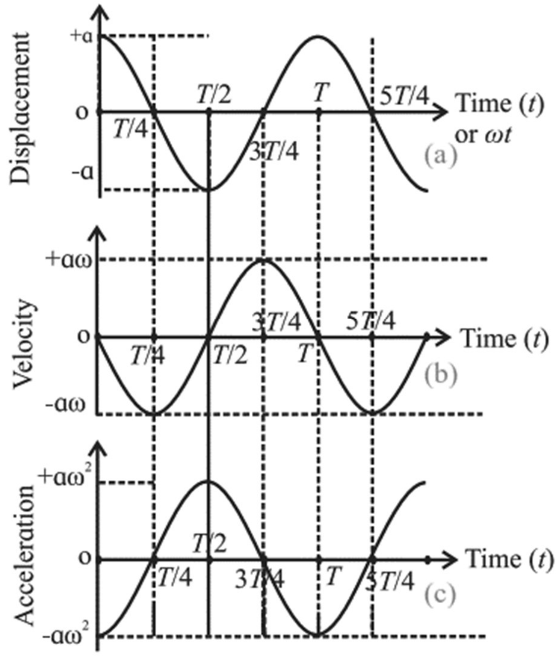
For acceleration, Centripetal acceleration acts along the radius and directed towards O. Its projection along diameter will be
a_y = – rω² sinθ
 = – rω²sin(ωt + ϕ)= – ω²y

Thus, projection of UCM along any diameter is a linear SHM

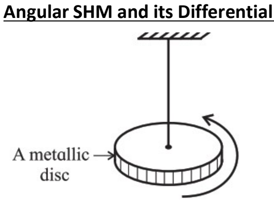
Graphical Representation of SHM:
Starting from mean position and going towards positive extreme



Starting from right/positive extreme



Angular SHM and its Differential Equation:



A metallic disc is centrally attached to a thin wire hanging from a rigid support. The disc is slightly twisted about the axis along the wire, and released. It starts to perform clockwise and anticlockwise rotational motion. Such oscillations are called angular or torsional oscillations.

Restoring torque will be directly proportional to the angular displacement

τ α – θ or τ = – cθ where c is constant of proportionality.

But, τ = I α where I is the moment of inertia and α is angular acceleration

Thus, I α = – c θ or I α + c θ = 0

Since $\alpha = \frac{d^2\theta}{dt^2}$,

$$thus\ \boldsymbol{I\,\frac{d^2\theta}{dt^2} + c\,\theta = 0}$$

Above is the differential equation for angular SHM

TIME PERIOD of angular SHM

$$T = \frac{2\pi}{\omega}$$

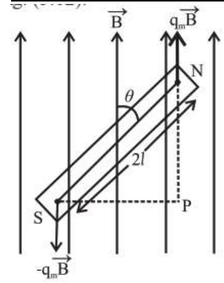
$$= \frac{2\pi}{\sqrt{angular\ displacement\ per\ unit\ angular\ displacement}} \quad \dots (I)$$

Since , I α = c θ , Thus, $\frac{\alpha}{\theta} = \frac{c}{I}$

Substitute in (I) we get

$$T = \frac{2\pi}{\sqrt{c/I}} = 2\pi\sqrt{\frac{I}{c}}\ where\ I\ is\ the\ moment\ of\ inertia$$

Magnet Vibrating in Uniform Magnetic Field:



If a bar magnet is freely suspended in a uniform magnetic field, then it aligns with is axis parallel to the magnetic field. If it is now given a small angular displacement θ and released, it performs angular oscillations.

Let, M (or μ or p) = dipole moment

= q. 2L

The restoring torque is τ = MBsin θ

If θ is small then, sin θ ≈ θ^c , Thus, τ =MBθ

But τ = Iα, where I:Moment of inertia of bar magnet and α : angular acceleration

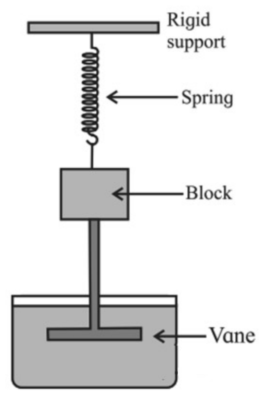
thus, I α = – MBθ or $\alpha = -\left(\frac{MB}{I}\right)\theta$

Since α is proportional to θ, the magnet is said to perform angular SHM

$$Time\ period\ T = \frac{2\pi}{\sqrt{angular\ displacement\ per\ unit\ angular\ displacement}}$$

$$= \frac{2\pi}{\sqrt{a/\theta}} = 2\pi\sqrt{\frac{I}{MB}}$$

DAMPED OSCILLATION



Periodic oscillations of gradually decreasing amplitude are called damped harmonic oscillations and the oscillator is called a damped harmonic oscillator.

A block mass m oscillates on a spring. A rod attached to it extends into a vane which is submerged into a liquid. As the vane moves up and down, a viscous drag is exerted by the liquid on the oscillating system.

Damping force F_d α – v where v is the speed of the vane and block. Thus, F_d = –bv, where b: damping constant and the negative sign shows the force is directed opposite the velocity. The spring force is F_s = –kx, where k: spring constant. Ignoring the gravitational force, we can write the total force as F = F_s + F_d

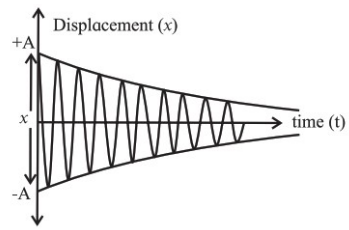
ma = –kx –bv , Thus, ma + bv + kx = 0

$$\boldsymbol{m\,\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0}$$

The solution is of the form

$$x = Ae^{\frac{-bt}{2m}}\cos(\omega' t + \phi)$$

$$\boldsymbol{\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} \quad and \quad T = \frac{2\pi}{\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}}}$$



As seen the amplitude decreases exponentially with time